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Pressure-fluctuation measurements on an oscillating circular cylinder

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Measurements are presented of the fluctuating pressure recorded at a point 90° from the mean position of the forward stagnation point on a circular cylinder oscillating in a water flow. The aspect ratio of the cylinder was 9·5 and the turbulence level in the free-stream was 5·5%. The cylinder Reynolds number was $2{\cdot}4 \times 10^4$ and the cylinder was forced to oscillate transverse to the main flow at amplitudes up to 1·33 cylinder diameters. The reduced velocity was varied over the range 3–18 and the experiments spanned the vortex-shedding lock-in range. Measurements of phase difference between pressure and displacement show that the maximum out-of-phase lift force occurs at an amplitude of about half a diameter. Good agreement is found between measurements on forced and freely oscillating cylinders. A simple potential-flow model gives reasonable predictions of the pressure fluctuations at the body frequency and at twice the body frequency at reduced velocities away from lock-in.

1. Introduction

One of the most common causes of flow-induced vibration of high aspect ratio bluff bodies is the regular shedding of vortices. Oscillations most frequently occur in a direction transverse to that of the mainstream and are often accompanied by large changes in the structure of the shed vortices: the correlation length, strength and formation distance can all be altered. In addition to the non-dimensional quantities that describe stationary-cylinder flow, oscillating-cylinder flow is sensitive to the amplitude ratio A/D, where A is the amplitude of oscillation and D is the cylinder diameter, and to the reduced velocity U/ND, where U is the flow velocity and N is the frequency of oscillation. If we compare the flows about a freely oscillating cylinder and a similar cylinder forced to oscillate, these will be the same if the Reynolds number, amplitude ratio and reduced velocity are the same for each case. This generalization needs qualification, however; for a forced cylinder A is held constant whereas for a freely vibrating cylinder there will be some modulation of the response, due to cycle-to-cycle variations in the shed vortices, and flow history will have an influence on wake structure. In the present investigation forced oscillations have been used since it is easier to control an experiment where A and N are held constant and there are less limitations on the range of A/D that can be examined.

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Numerous experimental and theoretical studies have been performed using the circular cylinder and this shape was chosen for the present investigation. However, in spite of the large volume of previous work, there are few measurements to be found of the lift forces acting on a circular cylinder forced to oscillate. Bishop & Hassan (1964) measured the fluctuating lift and drag forces on a circular cylinder forced to oscillate transverse to a water flow at Reynolds numbers within the subcritical regime. However, owing to problems involving the subtraction of inertia forces, caused by both the mass and added mass of the cylinder, their results are of limited accuracy. Nevertheless their findings are extremely interesting and their suggestion that the wake–cylinder combination behaves as a nonlinear oscillator initiated a productive line of theoretical study (see Hartlen & Currie 1970). Similar measurements have been made by Jones (1968) at Reynolds numbers in excess of $5{\cdot}5 \times 10^6$ but the maximum amplitude ratio tested was only 0·06. Such direct measurements of sectional forces are difficult to make accurately on a cylinder performing large amplitude oscillations and another possibility is to derive these forces from measurements of the fluctuating surface pressure. The fluctuating lift on stationary circular cylinders has been estimated by Loiseau & Szechenyi (1974), using 20 pressure transducers, and by Surry (1972), who used two transducers and made 157 cross-correlation measurements for each lift estimate. In the present investigation excessive cost ruled out the first method and excessive time the second. Instead it was decided to measure the fluctuating pressure at $\theta = 90°$, θ being measured from the mean position of the front stagnation point, and to assume that this has some characteristics in common with the fluctuating lift.

Although many authors have reported measurements of the fluctuating pressure on stationary circular cylinders there are very few measurements on oscillating cylinders. At the Reynolds number of the present investigation, approximately 2×10^4, Gerrard (1961), McGregor (1957), Feng (1968) and Novak & Tanaka (1975) have all measured pressure fluctuations on stationary cylinders. Measured values of C'_p, where C'_p is defined as the root-mean-square value of pressure fluctuations divided by the free-stream dynamic pressure, are sensitive to experimental conditions. They depend, amongst other things, upon the Reynolds number, cylinder surface finish, aspect ratio, end-plate design, tunnel turbulence level, blockage ratio and acoustic disturbances. Therefore it is not surprising that, at the centre-span and at $\theta = 90°$, the above authors show a spread in C'_p between 0·19 and 0·31, however the majority of values lie between 0·2 and 0·25. One might expect that when the cylinder is undergoing large amplitude oscillations many of these factors will lose their importance and C'_p will depend primarily on the cylinder vibration and Reynolds number. A few measurements of the fluctuating pressure on an oscillating circular cylinder at Reynolds numbers around 2×10^4 have been reported by Feng (1968) and Novak & Tanaka (1975). Feng's measurements were made on a freely vibrating cylinder over a range of amplitudes whereas Novak & Tanaka's were for a forced cylinder at $A/D = 0{\cdot}05$ for one unspecified reduced velocity.

The aim of the present investigation is to examine experimentally the fluctuating pressure at $\theta = 90°$ for a range of amplitude ratios and reduced velocities. The experiments are conducted in water, rather than air, because for a given Reynolds number, reduced velocity and cylinder size the frequency for vortex lock-in is proportional to the kinematic viscosity of the fluid and for simplicity of mechanical design one wishes to keep this frequency as low as possible. The results will be compared with Feng's measurements made under freely oscillating conditions. It is hoped that such results will provide some understanding of large amplitude vortex-induced vibrations and will be helpful in testing mathematical models.

2. Experimental arrangement

The experiments were conducted in an open water channel in the Department of Mechanical Engineering at the University of Toronto. The water channel was 0·91 m wide and 0·91 m high and the water depth could be varied by an adjustable gate at the downstream end. The overall length of the channel, from the end of the contraction to the downstream gate, was 12·8 m and the experiments were conducted at a station 8·63 m from the upstream end. Throughout the experiments the water depth was kept constant at 0·66 m and the flow rate was 0·26 m^3/s, giving a mean velocity across the channel of about 0·43 m/s. Two coarse grids were situated in the settling chamber in order to improve the uniformity of the mean velocity across the channel. The distribution of mean velocity in the empty channel was measured with a Pitot tube connected to an inclined manometer and the turbulence level was measured using a hot-film probe.

The circular cylinder used was made of Perspex and was 57·2 mm in diameter. The top of the cylinder projected through the water surface and there was a small clearance between the bottom of the cylinder and the floor of the channel. Thin circular plates 152 mm diameter were added at either end, one being 76 mm below the free surface and the other 40 mm above the floor of the channel. The aspect ratio of the cylinder was 9·5 between the end plates. A Setra model 237 pressure transducer with a pressure range from 0 to 1 psi was mounted inside the cylinder at a height equal to about half the water depth. The transducer was connected to a pressure hole in the surface of the cylinder by a 50 mm length of tubing about 1·3 mm internal diameter. The natural frequency of the transducer and tubing system, when filled with water, was in excess of 100 Hz. The cylinder occupied 6% of the cross-sectional area of the channel but the pressure measurements have not been corrected for the effects of blockage. There is some uncertainty about correcting fluctuating-pressure measurements but if blockage is considered to be simply an increase in stream velocity then the measurements of C_p' would be about 6% too high.

The cylinder was attached to a carriage mounted above the water surface and the carriage could be oscillated in a direction transverse to that of the flow by using a scotch-yoke mechanism. The mechanism was driven by a $\frac{1}{4}$ h.p. variable-speed d.c. motor and the frequency of cylinder oscillation could be varied continuously between 0 and 2·35 Hz. The amplitude of oscillation was adjustable in steps of 12·7 mm up to a maximum amplitude of 76·2 mm. This maximum amplitude corresponds to a peak-to-peak oscillation of 2·66 cylinder diameters. The displacement of the cylinder was monitored by a linear potentiometer. The natural frequency of the cantilevered cylinder was estimated to be an order of magnitude higher than the highest cylinder forcing frequency.

The pressure and displacement transducer signals were passed through two identical low pass filters set to pass all frequencies below 10 Hz. In order to prepare the signals for recording the d.c. levels were backed off against a d.c. supply. The signals were

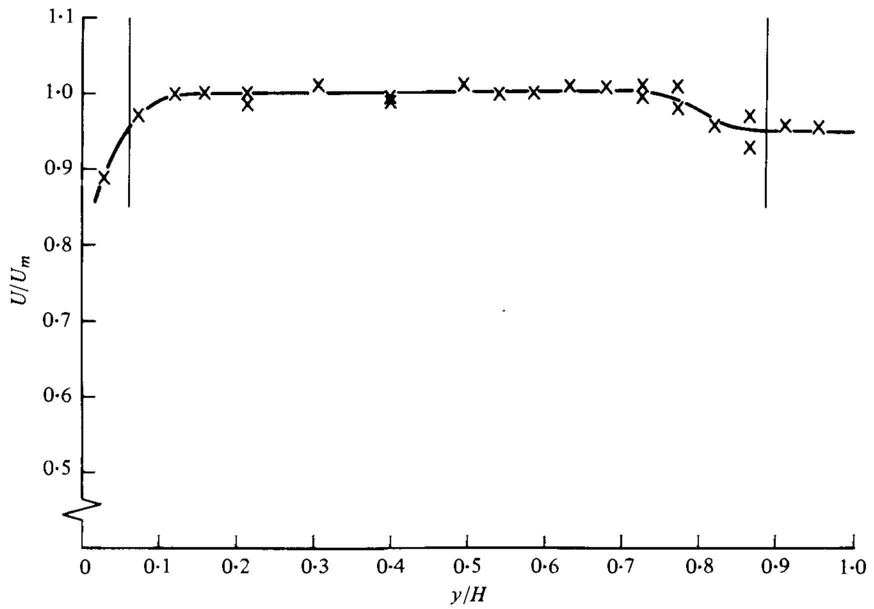

FIGURE 1. Vertical profile of mean velocity at the centre of the channel. |, end-plate position.

recorded simultaneously on two channels of a 14-channel Ampex PR 2200 analog tape recorder and, owing to the low frequencies involved, recording times of 20 min were used. The tape recorder was transported later to the Computer Research Facility of the University of Toronto and digitization and analysis of the signals carried out. Power spectra of the pressure and displacement signals were computed together with the cross-correlation between the two signals. The cross-correlation was used to calculate the phase difference between the displacement of the cylinder and that part of the pressure signal that was at the cylinder frequency.

3. Results

Measurements of the mean velocity in the channel at the test station were made with the aid of a Pitot tube. A vertical profile taken at the centre of the channel is shown in figure 1, where the local velocity U, non-dimensionalized by the mean velocity U_m across the central portion of the channel, is plotted against y/H, where y is the distance from the bottom of the channel and H is the water depth. A profile across the channel, taken at about the half-depth, is shown in figure 2, where z is the distance from one wall and W is the width of the channel. The average velocity at the test station was found to be 0·43 m/s. The error in measuring the velocity at any point was about $1\frac{1}{2}\%$ and therefore much of the scatter seen in figures 1 and 2 can be attributed to measurement error.

Figure 1 shows a reduction in the velocity towards the free surface and within the boundary layer on the channel floor. Indicated in the figure are the positions of the end plates that were added to improve the two-dimensionality of the flow around the cylinder. Inevitably their positioning was a matter of compromise between the degree of mean-flow uniformity and the aspect ratio of the cylinder and, reluctantly, some

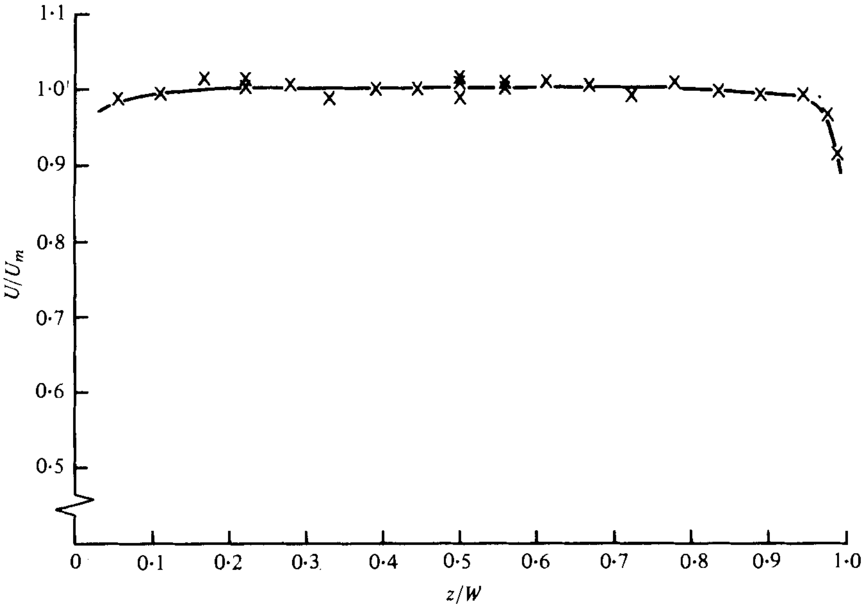

FIGURE 2. Horizontal profile of mean velocity at $y/H = 0{\cdot}49$.

degree of non-uniformity was accepted. All the fluctuating-pressure measurements were made at the half-depth, where the approaching flow was closely uniform. The profile of mean velocity across the channel plotted in figure 2 shows the mean flow to be uniform across 80% of the channel.

The fluctuating-pressure measurements were performed at a Reynolds number of $2{\cdot}4 \times 10^4$ and the experiments were carried out for a range of values of U/ND and A/D. The reduced velocity was varied by changing the oscillation frequency hence the Reynolds number, based on U, remained constant throughout. Initially the cylinder was held stationary and measurements of C_p' were made at $\theta = 0$ and $\theta = 90°$. The value of C_p' measured at $\theta = 90°$ was 0·22, which is in the middle of the range recorded by Feng (1968) at a similar Reynolds number. At $\theta = 0$, C_p' was found to be 0·11, which is an order of magnitude higher than the value measured by Feng (1968) and McGregor (1957). McGregor, in particular, showed that vortex shedding induces very small fluctuating pressures at the stagnation point and the high fluctuating pressures recorded in the present experiment are probably related to turbulence in the approaching flow. Bearman (1972) measured pressure fluctuations at the stagnation point on a bluff body and showed that, so long as the scale of turbulence is large compared with the width of the body, $C_p' \approx 2(\overline{u^2})^{\frac{1}{2}}/U$, where $(\overline{u^2})^{\frac{1}{2}}$ is the r.m.s. value of the velocity fluctuations in the mean-flow direction. This simple result follows from the theory of turbulent flow around bluff bodies by Hunt (1973). Therefore, assuming L_x/D is large, the fluctuating-pressure measurements suggest that the turbulence level in the channel at the test station could be as high as 5·5%. If the integral scale of the turbulence is large it follows that the power spectrum of approaching fluctuations can be deduced from the power spectrum of pressure fluctuations at the stagnation point (see Bearman 1972). From the value of the power spectral density at low wavenumbers, it is suggested that the integral scale of the turbulence is about

0·15 m, i.e. 2·6 times the diameter of the cylinder. At a later stage in the investigation, in order to measure the turbulence level more directly, a hot-film probe was used and this also recorded a turbulence level, for the longitudinal component, of 5·5%. It seems that this turbulence was generated in the settling chamber where the flow entered from the pump. Although the two very coarse grids in the settling chamber were successful at improving the mean velocity distribution they were not suitable for reducing turbulence. Within the time available to perform the experiments it was not practical to build an improved settling chamber so the results presented are for a free-stream turbulence level of 5·5%.

The addition of free-stream turbulence reduces the critical Reynolds number for a circular cylinder and Bearman (1968) has suggested that the beginning of the critical regime is a function of the Taylor parameter $[(\overline{u^2})^{\frac{1}{2}}/U]\,(D/L_y)^{\frac{1}{5}}$, where L_y is the lateral scale of the turbulence. The values of turbulence intensity and integral scale measured in this experiment suggest that the critical Reynolds number was about 7 or 8×10^4, i.e. about three times higher than the Reynolds number used for the measurements. Therefore it is expected that the cylinder flow was within the subcritical regime. The Strouhal number $S = nD/U$, where n is the vortex-shedding frequency, was estimated from the peak in the pressure-fluctuation spectrum measured at $\theta = 90°$ and this gave a value of 0·20. If blockage is taken into account in calculating U this reduces S to 0·19, which is the value expected in subcritical flow at this Reynolds number.

Ideally, when making pressure measurements on oscillating cylinders, the pressure transducer should be mounted flush with the cylinder surface. However the comparatively large size of a pressure-transducer diaphragm often means that it has to be mounted inside the cylinder and connected to a surface pressure hole by a length of tubing. If the model experiences any acceleration along a line connecting the transducer to the surface hole a pressure difference will develop between the hole and the face of the transducer. If this distance is h it can be shown that the coefficient of fluctuating pressure $C'_{pA}(N)$ is given by $2^{\frac{1}{2}}Ah(2\pi N)^2/U^2$, where $C'_{pA}(N)$ is defined in the same way as C'_p. In the present experiments h/D was 0·27 and at the highest value of A/D used, 1·33, the coefficient of fluctuating pressure due to this effect alone rises as high as 1·25 at $U/ND = 4$. Some calibration experiments were carried out with the pressure-transducer tubing filled with water but with the channel empty. Fluctuating pressures measured with the cylinder oscillating agreed very well with the predictions from the relation given above, and the pressure signal was in phase with the cylinder acceleration. Power spectra of the pressure and displacement signals measured under these conditions indicated only power at the oscillation frequency.

All the measurements of pressure fluctuations and phase angles between the cylinder displacement and pressure had to be corrected to take account of the effect of the length of tubing. The fluctuating lift on a cylinder is 180° out of phase with the fluctuation pressure at $\theta = 90°$ therefore phase angles quoted throughout this paper are for the phase by which suction on the cylinder leads the displacement. The measured r.m.s. value of the component of the fluctuating pressure at the cylinder oscillation frequency, $C'_{pT}(N)$, was estimated from the power spectrum of the total signal. The phase angle β for this signal was found from the cross-correlation between pressure and displacement. The coefficient $C'_p(N)$ of r.m.s. pressure fluctuations at frequency N that would be measured by a flush transducer can then be found from the relation

$$C'_p(N)\sin(2\pi Nt+\phi)+C'_{pA}(N)\sin(2\pi Nt) = C'_{pT}(N)\sin(2\pi Nt+\beta), \tag{1}$$

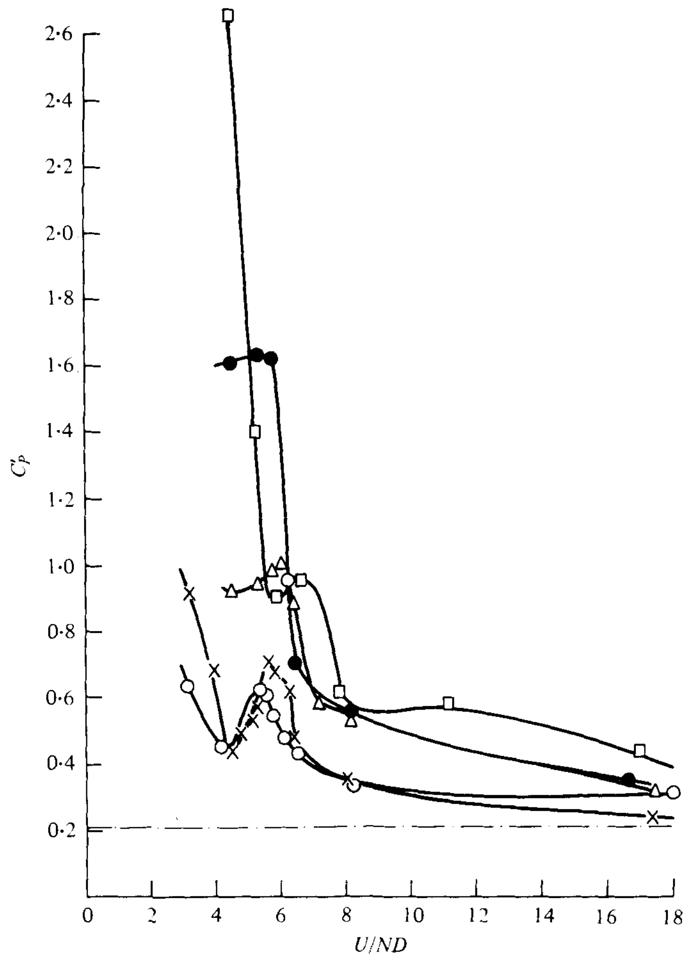

FIGURE 3. Pressure fluctuations at $\theta = 90°$ *vs.* reduced velocity. —·—, $A/D = 0$; ○, $A/D = 0{\cdot}11$; ×, $A/D = 0{\cdot}22$; △, $A/D = 0{\cdot}44$; ●, $A/D = 0{\cdot}89$; □, $A/D = 1{\cdot}33$.

where ϕ is the corrected phase angle,

$$\phi = \tan^{-1}\left(\frac{C'_{pT}(N)\sin\beta}{C'_{pT}(N)\cos\beta - C'_{pA}(N)}\right). \tag{2}$$

The corrected r.m.s. pressure coefficient C'_p is given by

$$C'_p = (C'_{pA}(N)^2 + C'^2_{pm} - 2C'_{pA}(N)\,C'_{pT}(N)\cos\beta)^{\frac{1}{2}}, \tag{3}$$

where C'_{pm} is the measured fluctuating pressure coefficient.

Measurements of C'_p at $\theta = 90°$ are shown in figure 3 plotted against reduced velocity for amplitudes up to $A/D = 1{\cdot}33$. The levels of C'_p rise as U/ND is reduced and in most cases show a peak in the range of vortex lock-in, somewhere between $U/ND = 5$ and 7. At the higher values of A/D this peak becomes much less distinct and appears as no more than a kink in the curve at $A/D = 1{\cdot}33$. C'_p rises rapidly at low values of U/ND as the pressure fluctuations become dominated by fluid acceleration effects. Presumably the curves for $A/D = 0{\cdot}44$ and $0{\cdot}89$ would also begin to rise if measurements

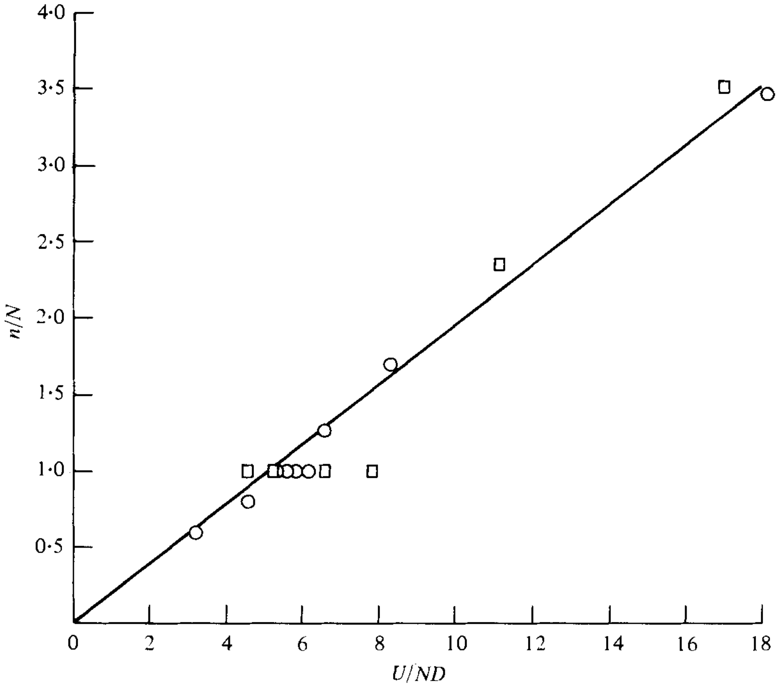

FIGURE 4. Vortex-shedding frequency *vs*. reduced velocity. ——, $A/D = 0$; ○, $A/D = 0{\cdot}11$; □, $A/D = 1{\cdot}33$.

were made at lower values of U/ND. The stationary-cylinder results are shown for comparison and it can be seen that at high values of U/ND, when the cylinder vibrates at a lower frequency than that at which vortices are naturally shed, the curves are approaching the stationary-cylinder value.

Measurements of the vortex-shedding frequency n are shown in figure 4 plotted against reduced velocity. For the sake of clarity only measurements for $A/D = 0{\cdot}11$ and 1·33 are shown. The straight line depicts the vortex-shedding frequency for the stationary cylinder and the oscillation frequency of the cylinder intersects this line at $U/ND = 5{\cdot}10$. At high values of U/ND, above say 15, the vortex-shedding frequency is found as a clear, pronounced peak in the pressure spectrum. When U/ND is reduced towards the lock-in value the natural shedding frequency peak weakens as power is drained out into the peak at the body frequency. As reported by previous authors, the extent of the lock-in range increases with amplitude, however a lower limit to lock-in is difficult to detect at large values of A/D owing to the dominance of inertia effects.

Figures 5 and 6 show, respectively, the variation of the coefficient $C_p'(N)$ of r.m.s. pressure fluctuations at the body frequency and the coefficient $C_p'(2N)$ at twice the body frequency plotted against reduced velocity for the range of amplitudes tested. The variation of $C_p'(N)$ follows closely the variation of C_p' shown in figure 3 since, except at high values of U/ND where the natural shedding frequency is the most conspicuous, the body frequency dominates the pressure spectrum. The next strongest frequency to emerge was the one at twice the body frequency and $C_p'(2N)$ generally increases with decreasing reduced velocity but, compared with $C_p'(N)$, there is a

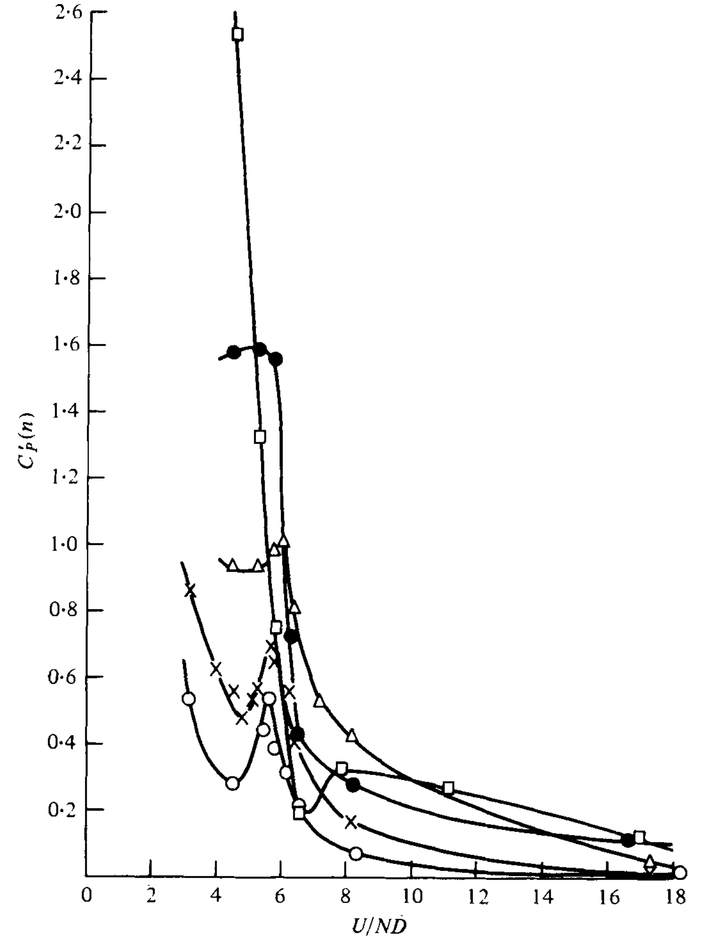

FIGURE 5. Pressure fluctuations at body frequency at $\theta = 90°$ *vs.* reduced velocity. Symbols as in figure 3.

greater dependence on amplitude at low values of U/ND. The signal at twice the body frequency shows a small peak within the lock-in range.

In order to determine whether the cylinder, if flexibly mounted, would be susceptible to flow-induced vibrations it is important to know the phase angle between the lift force on the cylinder and the displacement. The pressure at $\theta = 90°$ is strongly related to the lift and the phase angle between the suction at this point and the displacement is expected to be close to the phase angle between the lift and the displacement. The phase angle ϕ is plotted in figure 7 and it shows the expected large change in phase angle as U/ND is taken through the resonance with vortex shedding. For the lift force to be capable of exciting oscillations the phase angle needs to be within the range $0 < \phi < 180°$. At reduced velocities below lock-in the phase angle is near zero, confirming that the pressure is mainly related to the acceleration of the cylinder. At the highest reduced velocities examined the phase angle is shown for only the two largest amplitudes because at lower amplitudes $C'_p(N)$, as shown in figure 5, was too low to give a reliable estimate of ϕ.

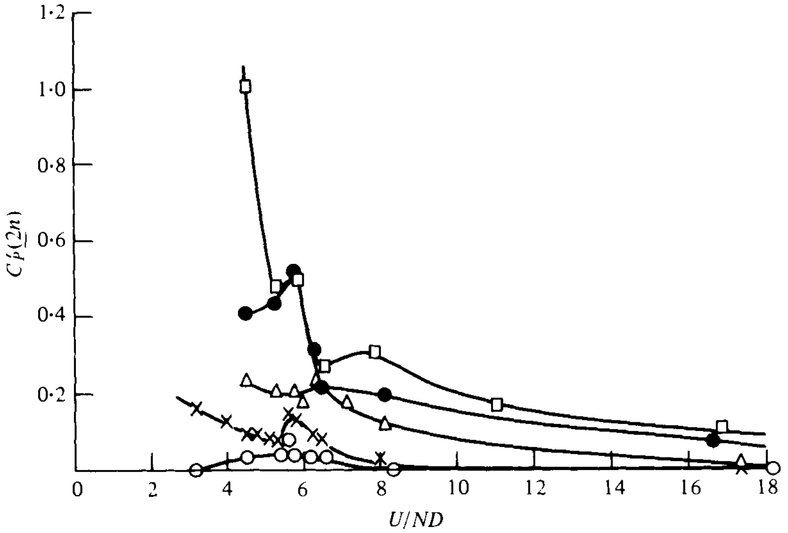

FIGURE 6. Pressure fluctuations at twice body frequency at $\theta = 90°$ *vs.* reduced velocity. Symbols as in figure 3.

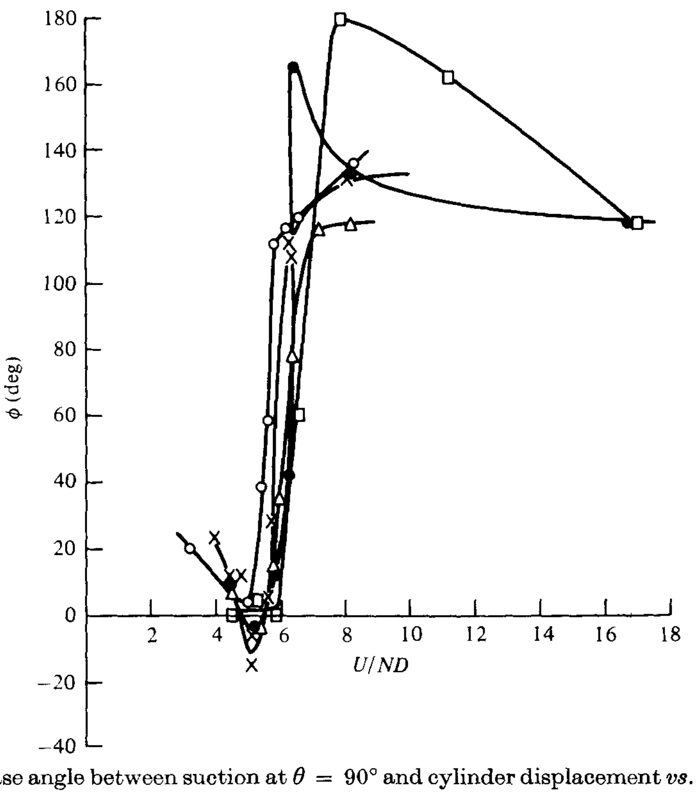

FIGURE 7. Phase angle between suction at $\theta = 90°$ and cylinder displacement *vs.* reduced velocity. Symbols as in figure 3.

Phase angles were measured, from the cross-correlation between pressure and displacement, to an accuracy of about 3 or 4°. When these results were corrected to take account of the effect of acceleration of the fluid within the tubing, according to (2), the accuracy of the resulting values of ϕ was hardly affected. This is because the fluctuating pressure coefficient $C'_{pA}(N)$ is proportional to $(ND/U)^2$ and attains a substantial value at only low reduced velocities, when it is in phase with the pressure acting on the cylinder surface. The pressure coefficient C'_{pm} could be measured to an accuracy of better than 2%. The effect of applying the correction given in (3) was to reduce the accuracy of the results at low values of U/ND to about 3%. It should be pointed out, however, that had no correction been applied C'_p would have been over 30% too high at the larger amplitudes for $U/ND = 4{\cdot}5$. The values of $C'_p(N)$ and $C'_p(2N)$ plotted in figures 5 and 6 were estimated from spectra measurements. At values of U/ND above lock-in the pressure fluctuations at the body frequency are comparatively weak and at $U/ND = 17$ their accuracy is estimated to be about 5%. At lock-in and below, the accuracy is the same as that for the total signal C'_p. The measurements of $C'_p(2N)$ required no correction but their accuracy is limited to about 5%.

4. Discussion of results

The similarity between forced and freely oscillating cylinder flows has been discussed in the introduction. Results from the present experiments can be compared with those of Feng (1968), who carried out measurements on a freely oscillating circular cylinder at approximately the same Reynolds number. Figure 8 shows measurements of the phase angle, between suction at $\theta = 90°$ and cylinder displacement, for a forced and a free cylinder and it can be seen that the agreement is good. Feng also presents a few measurements of C'_p for an oscillating cylinder and, where it is possible to compare results with the present work, agreement is satisfactory. These agreements suggest that the high turbulence level of the stream in the present experiments could have had only a small effect on the basic structure of the cylinder flow and probably this was negligible for the highest amplitudes of oscillation.

In order to calculate the response of flexibly mounted cylinders we need to know the magnitude of the part of the lift force that is out of phase with the motion. If this force is positive then oscillations will develop and the amplitude will increase until a balance is achieved between it and the structural damping forces. If again we assume the suction at $\theta = 90°$ to be characteristic of the lift then the out-of-phase force will be related to $C'_p(N)\sin\phi$. The maximum value of this term, for a given amplitude, has been calculated from the data plotted in figures 5 and 7 and, because of the rapid variation in phase through lock-in, it peaks very sharply between $U/ND = 5$ and 7. Maximum values of $C'_p(N)\sin\phi$ are plotted in figure 9 against amplitude and display a number of interesting features. At small amplitude fluid–structure interaction has the effect of increasing $C'_p(N)\sin\phi_{\max}$ dramatically. Studies by Davies (1975) on the wake of an oscillating D-section cylinder within the lock-in range show that small amplitude oscillations increase the strength of shed vortices and decrease the vortex formation distance. In addition, of course, we have the familiar increase in the spanwise correlation length when a body is oscillated as reported, for example, by Obasaju (1977).

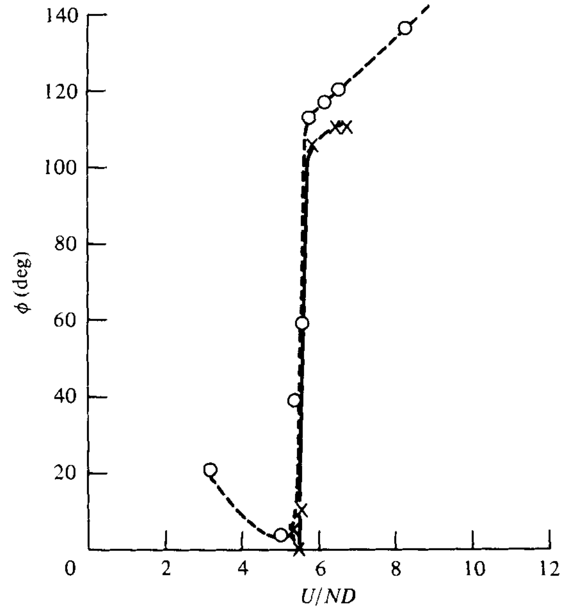

FIGURE 8. Phase-angle measurements on a forced and a freely vibrating cylinder at $A/D = 0{\cdot}11$. ○, forced cylinder; ×, free cylinder (Feng 1968).

The value of U/ND at the maximum value of $C'_p(N)\sin\phi$ is higher than that at which the stationary-body shedding frequency would coincide with the oscillation frequency. Therefore the resulting Strouhal number is always lower than the stationary cylinder value and this suggests that, comparatively, there is a longer time for the vortex sheet shed from an oscillating cylinder to roll up. Thus one might expect this to initiate stronger vortices and that these would develop more intense pressure fluctuations. At $A/D = 0{\cdot}44$ the maximum value of $C'_p(N)\sin\phi$ occurs at a Strouhal number 20% lower than the stationary-cylinder value.

Figure 9 shows that the value of U/ND for the maximum out-of-phase lift force progressively increases with increasing A/D for A/D up to at least 0·44. This suggests that hysteresis can exist in the response of flexibly mounted cylinders if the damping is low enough to allow large amplitude oscillations to build up. For a freely vibrating cylinder with fixed mechanical properties $A/D \propto (U/ND)^2 C'_L(N)\sin\phi$, where $C_L(N)$ is the coefficient of fluctuating lift at the frequency of oscillation (see, for example, Parkinson 1974). Therefore as the velocity increases A/D will increase but because of the fluid–structure interaction the increases in both A/D and U/ND can lead, as figure 9 indicates, to larger values of the out-of-phase lift force. A/D increases, therefore, until a new equilibrium is reached. However, if the velocity is reduced from a value above the lock-in range there is no continuous path to the large values of out-of-phase lift that can sustain the largest amplitudes. Compared with the case of increasing velocity, decreasing the velocity should show a lower peak amplitude occurring at a lower value of U/ND. Such a hysteresis is a prominent feature of Feng's results for a lightly damped circular cylinder.

It is interesting to note in figure 9 that the out-of-phase force reaches a maximum at an amplitude of roughly half a cylinder diameter. Extrapolating the data, we can

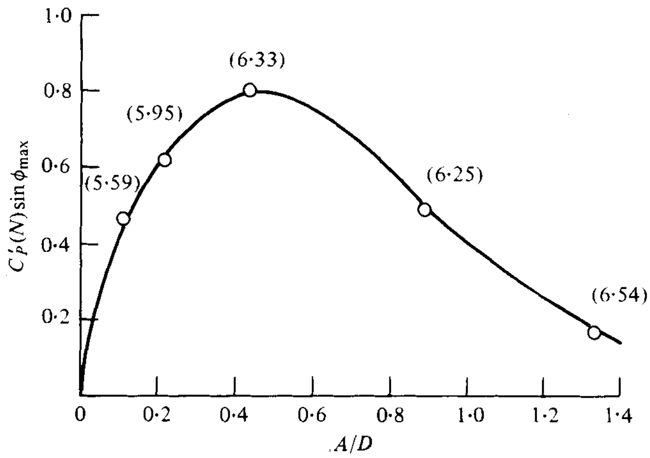

FIGURE 9. Maximum value of out-of-phase pressure fluctuation at $\theta = 90°$ *vs.* amplitude. Numbers in parentheses denote reduced velocity.

posulate that an undamped circular cylinder would not oscillate beyond an amplitude of between 1·5 and 2 diameters. At $A/D = 1{\cdot}33$ the maximum out-of-phase suction occurs at about $U/ND = 6{\cdot}5$ and inspection of figures 5 and 6 shows that $C_p'(N)$ is small and less even than $C_p'(2N)$; therefore there seems to be a breaking down of the basic vortex-shedding flow structure. During high amplitude oscillations the maximum velocity of the cylinder is of the same order as the flow velocity. The ratio of the maximum cylinder velocity to the stream velocity is given by $(2\pi A/D)(ND/U)$ and at $A/D = 1{\cdot}33$ and $U/ND = 6{\cdot}5$, for example, the maximum cylinder velocity is $1{\cdot}29U$. A pertinent question to ask, therefore, is how will the cylinder flow respond to such large fluctuations in the magnitude and direction of the relative approaching stream? Observing the free surface of the water, for the high amplitude cases, the flow around the cylinder appears to align itself with the instantaneous direction of the stream. The separation points move through large angular distances and the cylinder motion must be opposed by a component of the instantaneous drag force. These observations may be suspect because the cylinder was fitted with an end plate located just below the water surface. However, the excellent flow-visualization photographs by Meier-Windhorst (1939) of a freely vibrating cylinder executing large amplitude oscillations show similar very large movements of the separation position.

It is difficult to know how even to begin to develop a mathematical model of the flow around an oscillating circular cylinder which will describe all the effects that have been observed. Models using the nonlinear-oscillator concept have gone a long way towards predicting the form of the lift-force variation during lock-in for modest amplitudes of oscillation. In order to predict pressure fluctuations at reduced velocities away from lock-in, it was decided to explore the limitations of a simple potential-flow approach.

The instantaneous value $C_p(t)$ of the pressure coefficient for an oscillating circular cylinder at $\theta = 90°$ is given in potential flow by the expression

$$2\pi^2 \frac{A}{D}\left(\frac{ND}{U}\right)^2\left(\frac{A}{D}\cos 4\pi Nt - 2\sin 2\pi Nt\right).$$

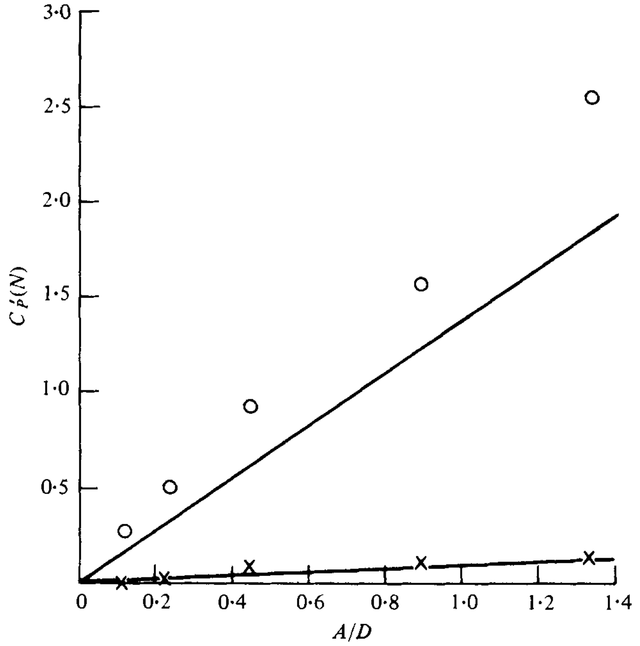

FIGURE 10. Pressure fluctuations at body frequency at $\theta = 90°$ *vs.* amplitude. ○, $U/ND = 4 \cdot 5$; ×, $U/ND = 16 \cdot 0$; ——, equation (4).

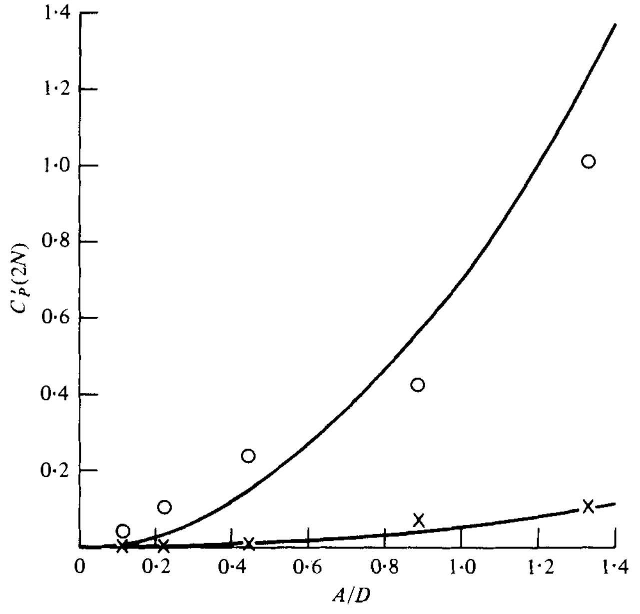

FIGURE 11. Pressure fluctuations at twice body frequency at $\theta = 90°$ *vs.* amplitude. ○, $U/ND = 4 \cdot 5$; ×, $U/ND = 16 \cdot 0$; ——, equation (5).

The first term, with twice the cylinder frequency, is generated by the changing incidence and relative velocity of the cylinder and the second term is related to the cylinder acceleration. Therefore it follows that

$$C'_p = 2^{\frac{1}{2}}\pi^2 \frac{A}{D}\left(\frac{ND}{U}\right)^2 \left|\left(\frac{A}{D}\right)^2 + 4\right|^{\frac{1}{2}},$$

$$C'_p(N) = 2^{\frac{3}{2}}\pi^2 \frac{A}{D}\left(\frac{ND}{U}\right)^2, \tag{4}$$

$$C'_p(2N) = 2^{\frac{1}{2}}\pi^2 \left(\frac{A}{D}\right)^2\left(\frac{ND}{U}\right)^2. \tag{5}$$

Measurements of $C'_p(N)$ at $\theta = 90°$ for a high and a low reduced velocity are plotted in figure 10 against A/D and compared with (4). Considering the grossness of the assumption made the agreement with experiment is surprisingly good and in particular the results show a linear variation of $C'_p(N)$ with A/D. Results for $C'_p(2N)$ are shown in figure 11 compared with (5) and agreement is equally good. In reality, of course, the flow will be a combination of the potential solution and the flow produced by the generation and shedding of vorticity. It appears that, away from lock-in, $C'_p(N)$ is related mainly to fluid inertia and that the inertia effects are reasonably described by the potential solution. This is a situation which also exists where bluff bodies are oscillated over similar amplitudes in still fluid.

By no stretch of the imagination would one have expected such a naive approach to work for $C'_p(2N)$. However there is some evidence from flow visualization to suggest that the flow, at large amplitudes, behaves in a quasi-steady manner. Therefore a component of the fluctuating pressure at $\theta = 90°$ will result from the quasi-steady changes in the pressure distribution. On a circular cylinder, in the subcritical regime the time-mean pressure coefficient beyond $\theta = 80°$ is nearly constant at about -1. Further forward, the mean C_p varies by about $\pm 20\%$ between $\theta = 80°$ and $50°$, and therefore over a large angular distance spanning $\theta = 90°$, C_p is approximately constant. This leads to a pressure fluctuation at twice the cylinder frequency equal to

$$2^{\frac{1}{2}}\pi^2 \left(\frac{A}{D}\right)^2\left(\frac{ND}{U}\right)^2 C_p^2.$$

If C_p is taken to be -1 this gives a result which is the same as that given by (5) except that the phase is 180° different. Clearly this suggests that the phase between $C'_p(2N)$ and the square of the cylinder displacement should be measured. The quasi-steady approach can be made more sophisticated by incorporating a better representation of the variation of C_p with θ. This leads to predictions of weaker pressure fluctuations at higher harmonics and these were visible in some of the spectra measurements.

5. Conclusions

Pressure fluctuations on the surface of a circular cylinder forced to oscillate in a water flow have been measured over a range of amplitudes up to $A/D = 1{\cdot}33$. The reduced velocity was varied over the range 3–18. At $\theta = 90°$, C_p rises as U/ND is reduced and shows a peak within the lock-in range. The maximum value of the fluctuations in suction at $\theta = 90°$, out of phase with the motion, occurs within the lock-in range and reaches a peak for $A/D \approx 0{\cdot}5$. It is predicted that the amplitude of an undamped cylinder would be limited to about two diameters. The measurements indicate the possibility of hysteresis in the response of freely vibrating cylinders similar to that observed by Feng (1968). The turbulence level in the approaching stream was 5·5% but, by comparison with Feng's measurements, this does not seem to have strongly influenced the results. Good agreement is found between Feng's measurements of the phase between the pressure and displacement on a freely oscillating cylinder and the phase measured here on a cylinder forced to oscillate. Away from lock-in, potential-flow modelling gives a surprisingly good prediction of the pressure fluctuations at the body frequency and at twice the body frequency. Fluctuations at the body frequency are mainly related to fluid inertia whereas those at twice the body frequency can be accounted for by quasi-steady changes in the pressure distribution. Agreement at twice the body frequency is shown to be fortuitous.

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